June 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/07, 8719/07

MATHEMATICS AND HIGHER MATHEMATICS
Paper 7 (Probability and Statistics 2)

UNIVERSITY *of* CAMBRIDGE
International Examinations

Page 1	Mark Scheme	Syllabus	Paper
	A AND AS LEVEL – JUNE 2004	9709/8719	7

1 (i) H_0: $\mu = 15$ or $p = 0.25$
H_1: $\mu > 15$ or $p > 0.25$

B1 — **1** — For H_0 and H_1 correct

(ii) Test statistic
$z = \pm\dfrac{21.5 - 15}{\sqrt{60 \times 0.25 \times 0.75}} = 1.938$

M1 — For attempt at standardising with or without cc, must have $\sqrt{\ }$ something with 60 in on the denom

OR test statistic
$z = \pm\dfrac{22/60 - 0.5/60 - 15/60}{\sqrt{\dfrac{0.25 \times 0.75}{60}}} = 1.938$

A1 — For 1.94 (1.938)

CV $z = 1.645$

M1 — For comparing with 1.645 or 1.96 if 2-tailed, signs consistent, or comparing areas to 5%

In CR Claim justified

A1ft — **4** — For correct answer(ft only for correct one-tail test)

2 (i) Mean = 3.5 + 2.9 + 3.1 = 9.5
Var = $0.3^2 + 0.25^2 + 0.35^2$ (=0.275)
St dev = 0.524

B1 — 9.5 as final answer
M1 — For summing three squared deviations
A1 — **3** — For correct answer

(ii) $z = \dfrac{9 - 9.5}{\sqrt{\dfrac{their\ var}{4}}} = -1.907$

M1 — For standardising, no cc

or $z = \dfrac{36-38}{\sqrt{(4 \times their\ var)}} = -1.907$

M1 — For $\sqrt{\dfrac{their\ var}{4}}$ or $\sqrt{(4 \times their\ var)}$ in denom - no 'mixed' methods.

$\Phi(1.907) = 0.9717 = 0.972$

A1 — **3** — For correct answer

3 (i) E(2X-3Y) = 2E(X) –3E(Y) = 16 – 18
= - 2

M1 — For multiplying by 2 and 3 resp and subt
A1 — **2** — For correct answer

(ii) Var (2X-3Y) = 4Var (X) +9Var (Y)
= 19.2 + 54

= 73.2

B1 — For use of var (Y) = 6
M1 — For squaring 3 and 2
M1 — For adding variances (and nothing else)
A1 — **4** — For correct final answer

4 (i) $\bar{x} = 375.3$
$\sigma^2{}_{n-1} = 8.29$

B1 — For correct mean (3.s.f)
M1 — For legit method involving n-1, can be implied
A1 — **3** — For correct answer

(ii) $p = 0.19$ or equiv.

B1 — For correct p

$0.19 \pm 2.055 \times \sqrt{\dfrac{0.19 \times 0.81}{200}}$

M1 — For correct form $p \pm z \times \sqrt{\dfrac{pq}{n}}$ either/both sides

B1 — For $z = 2.054$ or 2.055

$0.133 < p < 0.247$

A1 — **4** — For correct answer

Page 2	Mark Scheme	Syllabus	Paper
	A AND AS LEVEL – JUNE 2004	**9709/8719**	**7**

5 (i) $\dfrac{c-54}{3.1/\sqrt{10}}$ = -1.282	B1 M1		For + or – 1.282 seen For equality/inequality with their z ($\pm$) (must have used tables), no $\sqrt{10}$ needed (c can be numerical)
$c = 54 - 1.282 \times \dfrac{3.1}{\sqrt{10}}$ = 52.74	A1		For correct expression (c can be numerical, but signs must be consistent)
	A1	4	For correct GIVEN answer. No errors seen.
(ii) P($\bar{x}$ > 52.74) = 1 - $\Phi\left(\dfrac{52.74 - 51.5}{3.1/\sqrt{10}}\right)$	B1		For identifying the outcome for a type II error
	M1 A1		For standardising , no $\sqrt{10}$ needed For $\pm$ 1.265 (accept 1.26-1.27)
$= 1 - \Phi(1.265) = 1 - 0.8971$			
$= 0.103$ or 0.102	A1	4	For correct answer
6 (i) P(5) = $e^{-6} \times \dfrac{6^5}{5!}$ = 0.161	M1		For an attempted Poisson P(5) calculation, any mean
	A1	2	For correct answer
(ii) P($X \geq 2$) = 1 –{P(0) + P(1)} = 1 - $e^{-1.6}(1+1.6)$	B1 M1		For μ = 1.6, evaluated in a Poisson prob For $1 - P(0) - P(1)$ or $1 - P(0) - P(1) - P(2)$
= 0.475	A1	3	For correct answer
(iii) P(1 then 4 $\mid$ 5) = $\dfrac{\left(e^{-3} \times 3\right) \times \left(e^{-3} \times \dfrac{3^4}{4!}\right)}{e^{-6} \times \dfrac{6^5}{5!}}$	M1 M1		For multiplying P(1) by P(4) any (consistent) mean For dividing by P(5) any mean
	A1	3	For correct answer
= 0.156 or 5/32			
7 (i) $c\displaystyle\int_0^5 t(25 - t^2)\mathrm{d}t$ = 1	M1		For equating to 1 and a sensible attempt to integrate
$c\left[\dfrac{25t^2}{2} - \dfrac{t^4}{4}\right]_0^5 = 1$	A1		For correct integration and correct limits
$c\left[\dfrac{625}{2} - \dfrac{625}{4}\right] = 1 \Rightarrow c = \dfrac{4}{625}$	A1	3	For given answer correctly obtained
(ii) $\displaystyle\int_2^4 ct(25 - t^2)\mathrm{d}t = \left[\dfrac{25ct^2}{2} - \dfrac{ct^4}{4}\right]_2^4 = c[136] - c[46]$	M1* M1*dep		For attempting to integrate f(t) between 2 and 4 (or attempt 2 and 4) For subtracting their value when t = 2 from their value when t = 4
$= \dfrac{72}{125}$ (0.576)	A1	3	For correct answer
(iii) $\displaystyle\int_0^5 ct^2(25 - t^2)\mathrm{d}t = \left[\dfrac{4}{625} \times \dfrac{25t^3}{3} - \dfrac{4}{625} \times \dfrac{t^5}{5}\right]_0^5$	M1* A1 M1*dep		For attempting to integrate $tf(t)$, no limits needed For correct integrand can have c (or their c) For subtracting their value when t=0 from their value when t=5
$= \dfrac{8}{3}$	A1	4	For correct answer